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USE OF GENERALIZED DIFFUSION COEFFICIENTS IN SOLVING CONJUGATE PROBLEMS

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A numerical method is used to solve the conjugate problem of the heating of a graphite body in a high-temperature gas flow.

Calculation of the heating and loss of thermoprotective material when high-temperature gas (air, carbon dioxide, etc.) flows past an eroding surface involves the solution of a system of differential boundary-layer equations and the nonsteady heat-conduction equation for a solid. Consider the flow of a chemically reacting mixture in the vicinity of the forward critical point of a graphite body (Fig. 1). Steady laminar flow of thin mixture (consisting of ν elements and N components) is described by the following system of differential equations [1]: the continuity equation for the mixture

$$\frac{\partial}{\partial x}(\rho u r)+\frac{\partial}{\partial y}(\rho v r)=0; \tag{1}$$

the momentum equation

$$\rho u\frac{\partial u}{\partial x}+\rho v\frac{\partial u}{\partial y}=-\frac{dp}{dx}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right); \tag{2}$$

the diffusion equation for a chemical element

$$\rho u\frac{\partial \overline{c_\tau}}{\partial x}+\rho v\frac{\partial \overline{c_\tau}}{\partial y}+\frac{\partial \overline{K}_\tau}{\partial y}=0; \tag{3}$$

$$(\tau=1,\ 2,\ \ldots,\ \nu-1),$$

where

$$\overline{c_\tau}=\sum_{i=1}^{N}n_{\tau i}M_\tau c_i/M_i;\ \overline{K}_\tau=\sum_{i=1}^{N}n_{\tau i}M_\tau K_i/M_i;$$

the equation of thermochemical equilibrium for a reaction of the type

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$$A_i \rightleftharpoons \sum_{j=1}^{l} \beta_{ij} A_j, \tag{4}$$

$$K_{p,i} = \frac{\prod_{j=1}^{l} \bar{c}_j^{\beta_{ij}}}{\bar{c}_i} (pM)^{\Delta n}$$

$$(i = 1, 2, \ldots, N - \nu),$$

where

$$\bar{c}_i = c_i/M_i; \ \Delta n = \sum_{j=1}^{l} \beta_{ij} - 1;$$

the energy equation

$$\rho u c_{p_{eff}} \frac{\partial T}{\partial x} + \rho v c_{p_{eff}} \frac{\partial T}{\partial y} = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) - \frac{\partial}{\partial y}\left(\sum_{i=1}^{N} I_i K_i\right); \tag{5}$$

and the Stefan–Maxwell equation for the diffusional mass flows disregarding thermo- and barodiffusion

$$\frac{\partial x_i}{\partial y} = \frac{M}{\rho}\left[x_i \sum_{j \neq i}^{N} \frac{K_j}{M_j \cdot D_{ij}(1)} - \frac{K_i}{M_i} \sum_{j \neq i}^{N} \frac{x_j}{D_{ij}(1)}\right] \tag{6}$$

$$(i = 1, 2, \ldots, N - 1).$$

Closure of the system in equations (6) is achieved using the equation for the sum of vector flows

$$\sum_{i=1}^{N} \vec{K}_i = 0; \tag{7}$$

the Dalton equation

$$\sum_{\tau=1}^{\nu} \bar{c}_\tau = 1; \tag{8}$$

the equation of state

$$p = \rho \tilde{R} T/M, \tag{9}$$

where

$$M = \left(\sum_{i=1}^{N} \bar{c}_i\right)^{-1}; \tag{10}$$

and the heat-conduction equation for the body

$$\rho_b c_b \frac{\partial T_b}{\partial \tau} = \frac{\partial}{\partial y_1}\left(\lambda_b \frac{\partial T_b}{\partial y_1}\right). \tag{11}$$

The conjugate problem for the heating of the material is solved with the following initial and boundary conditions:

$$\text{when } \tau = 0 \quad T_b = T_0; \tag{12}$$

$$y = 0 \quad u = v = 0; \ \bar{K}_{\tau,w} = 0;$$

$$\lambda_w \left(\frac{\partial T}{\partial y}\right)_w - \sum_{i=1}^{N} (I_i K_i)_w - \varepsilon_w \sigma_0 T_w^4 = -\lambda_{b,w}\left(\frac{\partial T_b}{\partial y_1}\right)_w, \tag{13}$$

$$\text{as } y \to \infty \quad u \to u_e;\ T \to T_e, \tag{14}$$

$$\text{as } y_1 \to \infty \quad T_b \to T_0. \tag{15}$$

If several elements are present (ν = 3–5), when the number of mixture components may reach 20 or 30, the system in equations (1)–(11) with the initial and boundary conditions in Eqs. (12)–(15) is very complex. In practice, various assumptions and models for the diffusional mass flows are very often used for the computer realization of such complex systems [2, 3].

The present work employs the method of generalized diffusion coefficients [1], according to which the diffusional mass flows are written in the explicit form

$$K_i = -\rho D_i \frac{\partial c_i}{\partial y}, \tag{16}$$

where the generalized diffusion coefficient D_i is given by the expression

$$D_i = \frac{G}{\beta A_i}$$

$$\times \left\{1 + \left[\frac{\beta_i}{M} \sum_{j=1}^{N} \left(\frac{M_j}{A_j} \bar{c}_j\right) - \bar{c}_i\right] M \sum_{k \neq i}^{N} \frac{\partial \bar{c}_k}{\partial y} - \frac{\beta_i}{M} \sum_{j \neq i}^{N} \left(\frac{M_j}{A_j} \cdot \frac{\partial c_j}{\partial y}\right)\right\}; \tag{17}$$

$$\beta = \sum_{j=1}^{N} x_j A_j;\ \beta_i = x_i A_i;$$

when

$$0.2 \leqslant (M_i/M_j) \leqslant 5;$$

$$A_i = \sigma_i \left(\frac{\varepsilon_i}{k}\right)^{0.0815} M_i^{0.25};\ A_j = \sigma_j \left(\frac{\varepsilon_j}{k}\right)^{0.0815} M_j^{0.25}; \tag{18}$$

and when

$$(M_i/M_j) > 5$$

$$A_i = \sigma_i \left(\frac{\varepsilon_i}{k}\right)^{0.0815} M_i^{0.25};$$

$$A_j = \sigma_j \left(\frac{\varepsilon_j}{k}\right)^{0.0815} M_j^{0.25} \sqrt{2} \left(\frac{M_j}{M_i}\right)^{0.25}.$$

The mixture parameter G is

$$G = 2.38 \cdot 10^{-7}\, T^{1.663}/p. \tag{19}$$

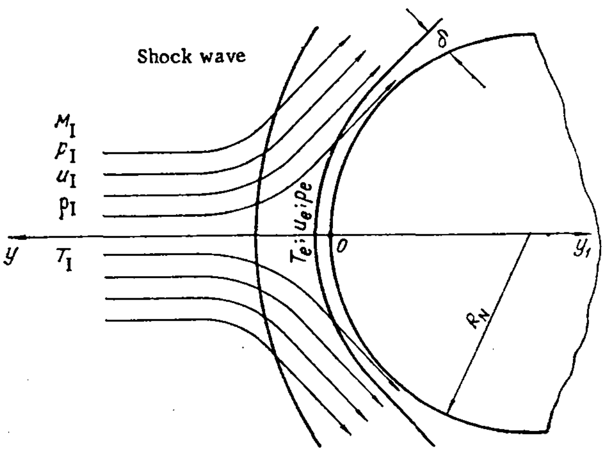

Fig. 1. High-temperature gas flow in vicinity of forward critical point of body.

TABLE 1. Comparison of Results for Diffusional Flow Obtained by Accurate and Approximate Methods

T (°K)	$2 \cdot 10^3$	$4 \cdot 10^3$	$6 \cdot 10^3$
$(\bar{K}_{N_2})$ acc	$0{,}926 \cdot 10^{-9}$	$0{,}468 \cdot 10^{-8}$	$0{,}387 \cdot 10^{-7}$
$(\bar{K}_{N_2})$ app	$0{,}90 \cdot 10^{-9}$	$0{,}460 \cdot 10^{-8}$	$0{,}394 \cdot 10^{-7}$
$(\bar{K}_{O_2})$ acc	$0{,}109 \cdot 10^{-8}$	$0{,}408 \cdot 10^{-7}$	$0{,}726 \cdot 10^{-10}$
$(\bar{K}_{O_2})$ app	$0{,}108 \cdot 10^{-8}$	$0{,}107 \cdot 10^{-7}$	$0{,}726 \cdot 10^{-10}$
$(\bar{K}_{NO})$ acc	$-0{,}177 \cdot 10^{-8}$	$0{,}438 \cdot 10^{-8}$	$0{,}12 \cdot 10^{-8}$
$(\bar{K}_{NO})$ app	$-0{,}178 \cdot 10^{-8}$	$0{,}438 \cdot 10^{-8}$	$0{,}12 \cdot 10^{-8}$
$(\bar{K}_{H_2})$ acc	$+0{,}794 \cdot 10^{-10}$	$0{,}394 \cdot 10^{-8}$	$0{,}414 \cdot 10^{-10}$
$(\bar{K}_{H_2})$ app	$+0{,}798 \cdot 10^{-10}$	$0{,}396 \cdot 10^{-8}$	$0{,}414 \cdot 10^{-10}$
$(\bar{K}_{N})$ acc	$-0{,}823 \cdot 10^{-15}$	$-0{,}691 \cdot 10^{-9}$	$-0{,}465 \cdot 10^{-7}$
$(\bar{K}_{N})$ app	$-0{,}824 \cdot 10^{-15}$	$-0{,}692 \cdot 10^{-9}$	$-0{,}472 \cdot 10^{-7}$
$(\bar{K}_{O})$ acc	$-0{,}174 \cdot 10^{-9}$	$-0{,}163 \cdot 10^{-7}$	$0{,}409 \cdot 10^{-8}$
$(\bar{K}_{O})$ app	$-0{,}174 \cdot 10^{-9}$	$-0{,}163 \cdot 10^{-7}$	$0{,}410 \cdot 10^{-8}$
$(\bar{K}_{H})$ acc	$-0{,}148 \cdot 10^{-9}$	$-0{,}681 \cdot 10^{-8}$	$0{,}241 \cdot 10^{-8}$
$(\bar{K}_{H})$ app	$-0{,}149 \cdot 10^{-9}$	$-0{,}687 \cdot 10^{-8}$	$0{,}242 \cdot 10^{-8}$

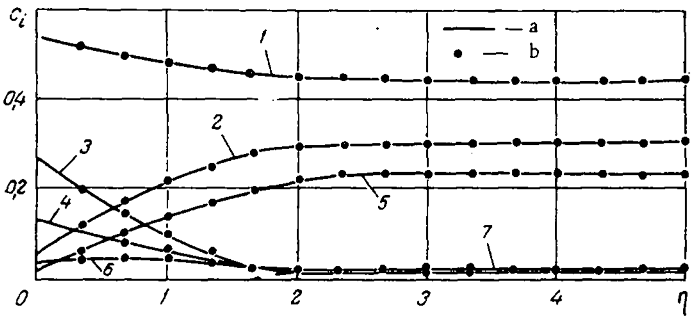

Fig. 2. Distribution of mixture-component mass concentrations across the boundary layer: 1) c_{N_2}; 2) c_H; 3) c_{H_2}; 4) c_{O_2}; 5) c_O; 6) c_{NO}; 7) c_N; a) accurate solution; b) approximate solution.

The heating of a graphite sphere is calculated over a wide range of the external flow parameters: $T_e = (5\text{-}8) \cdot 10^3$°K, $p_e = 1\text{-}100$ bar. The external flow is a mixture of air and hydrogen ($\bar{c}_N = 0.537$; $\bar{c}_O = 0.163$; $\bar{c}_H = 0.3$), and the multicomponent gas mixture consists of seven components (N_2; O_2; NO; H_2; N; O; H) differing in molecular weight by an order of magnitude. Numerical calculations are carried out using the accurate system in Eqs. (1)-(11) with the initial and boundary conditions in Eqs. (12)-(15) and by the approximate method using generalized diffusion coefficients, as in Eqs. (16)-(19).

Table 1 gives results for the normalized diffusional flow $\bar{K}_i = K_i/(\partial T/\partial y)$ obtained from Eqs. (6) and (16) for p = 1 bar.

It is evident from Table 1 that the generalized-coefficient method gives good accuracy for the diffusional flows. Over the whole temperature range the discrepancy with the accurate results was no more than 3-4% for all the components.

In Fig. 2, concentration profiles (c_i) across the boundary layer are given for all the components, together with results obtained by the approximate method. There is evidently good agreement. Similarly, there is good agreement for the temperature profile (Fig. 3), both in the boundary layer and in the body.

Thus, the generalized-coefficient method may successfully be used in calculating heating and the loss of thermoprotective material.

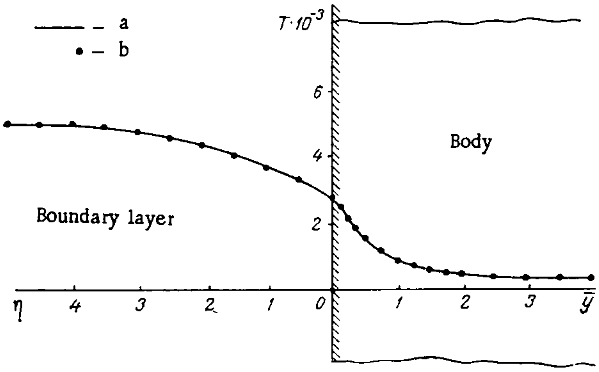

Fig. 3. Temperature profile in conjugate problem (gas—body): a) accurate solution; b) approximate solution. T, °K.

NOTATION

x, y, coordinates; u, v, velocity components; ρ, density; p, pressure; $\bar{c}_T$, element concentrations; c_i, mass concentrations of components; x_i, molar concentrations of components; $\bar{K}_T$, diffusional mass flows of elements; K_i, diffusional mass flows of components; T, temperature; $c_{p_{eff}}$, total specific heat of mixture; M_T, molecular weights of elements; M_i, molecular weights of components; M, molecular weight of mixture; $\tilde{R}$, universal gas constant; $D_{ij}(1)$, diffusion coefficient of binary mixture; D_i, generalized diffusion coefficients; μ, viscosity; λ, heat conduction.

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CONJUGATE HEAT-EXCHANGE PROBLEM IN THE FLOW OF A STREAM OF DISSOCIATED AIR OVER A BLUNT AXISYMMETRIC BODY

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An algorithm is constructed and the results of a numerical solution are presented for the conjugate problem of nonsteady heat exchange in the vicinity of the critical point of a blunt axisymmetric body during its interaction with a hypersonic airstream.

The nonsteady thermal interaction of an oncoming stream of liquid or gas with a solid body is characterized by the fact that the thermal boundary conditions at the surface over which the flow occurs vary with time. And these conditions are not known in advance but must be found in the course of the solution of the problem of nonsteady heat exchange.

The most general approach to the solution of problems of nonsteady convective heat exchange in a gas—solid body system consists in treating them as conjugate [1, 2]. A system of equations consisting of the equations for the nonsteady boundary layer for the gaseous zone and the heat-conduction equation for the solid

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